

Math 0308 Final Exam Review (answers)

Solve the given equations.

1. $3x + 14 = 8x - 1$

$$15 = 5x$$

$$3 = x$$

$$\boxed{3}$$

2. $3(4x - 1) = 5x + 6$

$$12x - 3 = 5x + 6$$

$$7x = 9$$

$$x = \frac{9}{7}$$

$$\boxed{\frac{9}{7}}$$

3. $8 - (3x + 4) = 6x$

$$8 - 3x - 4 = 6x$$

$$4 = 9x$$

$$\frac{4}{9} = x$$

$$\boxed{\frac{4}{9}}$$

4. $5(2x + 1) - 3(x + 4) = -21$

$$10x + 5 - 3x - 12 = -21$$

$$7x - 7 = -21$$

$$7x = -14$$

$$x = -2$$

$$\boxed{-2}$$

5. $3(4x + 5) - 5(2x - 7) = 2(6x + 1)$

$$12x + 15 - 10x + 35 = 12x + 2$$

$$2x + 50 = 12x + 2$$

$$48 = 10x$$

$$\frac{48}{10} = x$$

$$\boxed{\frac{24}{5}}$$

6. $2(3x + 1) + 4(2x - 2) = 2(7x - 3)$ 7. $5[2(3x - 1) - 6(x + 2)] = 3x - 7$

$$6x + 2 + 8x - 8 = 14x - 6$$

$$14x - 6 = 14x - 6$$

$$-6 = -6$$

$$\boxed{\text{all real numbers}}$$

$$5[6x - 2 - 6x - 12] = 3x - 7$$

$$5 \cdot -14 = 3x - 7$$

$$-70 = 3x - 7$$

$$-63 = 3x$$

$$-21 = x$$

$$\boxed{-21}$$

8. $3x + 14 = 3(x - 2) + 20$

$$3x + 14 = 3x - 6 + 20$$

$$3x + 14 = 3x + 14$$

$$14 = 14$$

$$\boxed{\text{all real numbers}}$$

9. $3x + 14 = 5(x - 2) - 2(x + 7)$

$$3x + 14 = 5x - 10 - 2x - 14$$

$$3x + 14 = 3x - 24$$

$$14 = -24$$

$$\boxed{\text{no solution}}$$

Solve the following inequalities. Graph your solution on a number line.

10. $x - 7 \leq -4$

$$x \leq 3$$



11. $4x + 9 > 3$

$$4x > -6$$

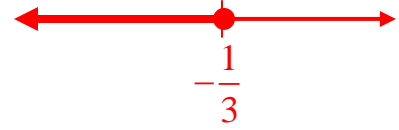
$$x > -\frac{3}{2}$$



12. $1 - 9x \geq 4$

$$-9x \geq 3$$

$$x \leq -\frac{1}{3}$$



13. $2x + 15 > 7x - 1$

$$-5x > -16$$

$$x < \frac{16}{5}$$



14. $-1 < 3x + 8 \leq 5$

$$-9 < 3x \leq -3$$

$$-3 < x \leq -1$$



15. $-4 < 4 - 2x < 6$

$$-8 < -2x < 2$$

$$4 > x > -1$$

$$-1 < x < 4$$



Use equations and algebraic methods to find solutions of the following problems.

16. Five minus 4 times a number is 3. What is the number?

$$5 - 4x = 3$$

$$-4x = -2$$

$$x = \frac{1}{2}$$

$$\frac{1}{2}$$

17. Find three consecutive odd integers whose sum is -21 .

$$x + (x + 2) + (x + 4) = -21$$

$$3x + 6 = -21$$

$$3x = -27$$

$$x = -9$$

$$-9, -7, -5$$

18. A 43 inch board is cut into three pieces. The second piece is one-half as long as the first piece. The third piece is 7 inches longer than the second piece. How long is each piece?

$$x + \left(\frac{1}{2}x\right) + \left(\frac{1}{2}x + 7\right) = 43$$

$$2x + 7 = 43$$

$$2x = 36$$

$$x = 18$$

$$\boxed{18", 9", 16"}$$

19. The perimeter of a rectangle is 36 feet. The length is 2 feet more than 3 times the width. What are the dimensions of the rectangle?

L is the length; *W* is the width.

$$2L + 2W = 36$$

$$L = 2 + 3W$$

$$2(2 + 3W) + 2W = 36$$

$$4 + 6W + 2W = 36$$

$$8W + 4 = 36$$

$$8W = 32$$

$$\boxed{W = 4'}$$

$$\boxed{L = 14'}$$

20. Michael has \$7.95 in dimes and quarters. He has a total of 39 coins. How many of each type of coin does he have?

D is the number of dimes; *Q* is the number of quarters.

$$10D + 25Q = 795$$

$$D + Q = 39$$

$$10(39 - Q) + 25Q = 795$$

$$390 - 10Q + 25Q = 795$$

$$15Q + 390 = 795$$

$$15Q = 405$$

$$\boxed{Q = 27}$$

$$\boxed{D = 12}$$

21. Jeffrey has \$11.75 in nickels and quarters. The number of quarters is 5 more than 4 times the number of nickels. How many of each type of coin does he have?

N is the number of nickels; *Q* is the number of quarters.

$$5N + 25Q = 1175$$

$$Q = 5 + 4N$$

$$5N + 25(5 + 4N) = 1175$$

$$5N + 125 + 100N = 1175$$

$$105N + 125 = 1175$$

$$105N = 1050$$

$$N = 10$$

$$Q = 45$$

22. Melissa has \$220 in \$1 bills, \$5 bills, and \$10 bills. The number of \$1 bills is 3 times the number of \$5 bills. The number of \$10 bills is 4 more than the number of \$5 bills. How many of each type of bill does she have?

O is the number of ones; *F* is the number of fives; *T* is the number of tens.

$$O + 5F + 10T = 220$$

$$O = 3F$$

$$T = 4 + F$$

$$3F + 5F + 10(4 + F) = 220$$

$$8F + 40 + 10F = 220$$

$$18F + 40 = 220$$

$$18F = 180$$

$$F = 10$$

$$O = 30$$

$$T = 14$$

Using the laws of exponents, simplify the following expressions. Write your answers with positive exponents. Assume that all variables represent non-zero numbers.

23. $\frac{3^6}{3^4}$

$$3^2$$

$$\boxed{9}$$

24. $(x^4)^6$

$$\boxed{x^{24}}$$

25. $\left(\frac{3x^4}{y^8}\right)^3$

$$\frac{3^3 x^{12}}{y^{24}}$$

$$\boxed{\frac{27x^{12}}{y^{24}}}$$

26. $\frac{39x^5 y^{12}}{3x^4 y^6}$

$$\boxed{13xy^6}$$

$$27. \left(\frac{8x^5y^6}{2x^2y^2} \right)^3$$

$$(4x^3y^4)^3$$

$$\boxed{64x^9y^{12}}$$

$$28. 5^0 + 5^{-1}$$

$$1 + \frac{1}{5}$$

$$\boxed{\frac{6}{5}}$$

$$29. x^4 \cdot x^{-7}$$

$$x^{-3}$$

$$\boxed{\frac{1}{x^3}}$$

$$30. \frac{x^{-5}}{x^2}$$

$$\frac{1}{x^5 \cdot x^2}$$

$$\boxed{\frac{1}{x^7}}$$

$$31. \frac{14x^{-2}y^6}{21x^{-10}y^{-3}}$$

$$\frac{2x^{10}x^{-2}y^6y^3}{3}$$

$$\boxed{\frac{2x^8y^9}{3}}$$

$$32. (4x^{-2}y^{-7})^{-3}(6^{-1}x^3y^{-2})^{-2}$$

$$4^{-3}x^6y^{21} \cdot 6^2x^{-6}y^4$$

$$\frac{6^2y^{25}}{4^3}$$

$$\frac{36y^{25}}{64}$$

$$\boxed{\frac{9y^{25}}{16}}$$

$$33. \left(\frac{4x^{-4}y^{-3}}{7x^2y^{-6}} \right)^{-2}$$

$$\left(\frac{4y^{-3}y^6}{7x^2x^4} \right)^{-2}$$

$$\left(\frac{4y^3}{7x^6} \right)^{-2}$$

$$\frac{4^{-2}y^{-6}}{7^{-2}x^{-12}}$$

$$\boxed{\frac{49x^{12}}{16y^6}}$$

$$34. \frac{(3x^{-8}y^{-2})(2x^{-2}y^{-1})^{-3}}{(8x^{-2}y)^{-2}}$$

$$\frac{3x^{-8}y^{-2} \cdot 2^{-3}x^6y^3}{8^{-2}x^4y^{-2}}$$

$$\frac{3 \cdot 8^2 \cdot x^{-2}y}{2^3x^4y^{-2}}$$

$$\frac{3 \cdot 64 \cdot yy^2}{8x^4x^2}$$

$$\boxed{\frac{24y^3}{x^6}}$$

Perform the indicated operations, and simplify all answers.

$$35. (x^2 - 5x - 12) + (7x^2 - 3x + 4)$$

$$\boxed{8x^2 - 8x - 8}$$

$$36. (5x^2 - x + 6) - (x^2 - 8x - 2)$$

$$\boxed{4x^2 + 7x + 8}$$

$$37. (-5x^3)(8x^7)$$

$$\boxed{-40x^{10}}$$

$$38. (3x + 8)(6x - 5)$$

$$\boxed{18x^2 + 33x - 40}$$

$$39. (5x + 6)(3x^2 - 2x + 3)$$

$$15x^3 - 10x^2 + 15x + 18x^2 - 12x + 18$$

$$\boxed{15x^3 + 8x^2 + 3x + 18}$$

$$40. (x - 11y)(x + 11y)$$

$$\boxed{x^2 - 121y^2}$$

$$41. (3x+5)^2$$

$$\boxed{9x^2 + 30x + 25}$$

$$42. (4x-3y)^2$$

$$\boxed{16x^2 - 24xy + 9y^2}$$

$$43. \frac{12x^2 - 40x + 28}{4x}$$

$$\frac{12x^2}{4x} - \frac{40x}{4x} + \frac{28}{4x}$$

$$\boxed{3x - 10 + \frac{7}{x}}$$

$$44. \frac{24x^5yz + 6x^4y^2z^3 - 4x^3y^3z^5}{3x^4yz^2}$$

$$\frac{24x^5yz}{3x^4yz^2} + \frac{6x^4y^2z^3}{3x^4yz^2} - \frac{4x^3y^3z^5}{3x^4yz^2}$$

$$\boxed{\frac{8x}{z} + 2yz - \frac{4y^2z^3}{3x}}$$

$$45. \frac{6x^2 + 22x - 5}{2x+8}$$

$$\boxed{3x - 1 + \frac{3}{2x+8}}$$

$$\begin{array}{r} 2x+8 \overline{) 6x^2 + 22x - 5} \\ \underline{-(6x^2 + 24x)} \\ -2x - 5 \\ \underline{-(-2x - 8)} \\ 3 \end{array}$$

$$46. \frac{2x^4 + 8x^3 + 7x^2 - 7}{2x^2 + 2x - 5}$$

$$\boxed{x^2 + 3x + 3 + \frac{9x+8}{2x^2+2x-5}}$$

$$\begin{array}{r} 2x^2 + 2x - 5 \overline{) 2x^4 + 8x^3 + 7x^2 - 7} \\ \underline{-(2x^4 + 2x^3 - 5x^2)} \\ 6x^3 + 12x^2 - 7 \\ \underline{-(6x^3 + 6x^2 - 15x)} \\ 6x^2 + 15x - 7 \\ \underline{-(6x^2 + 6x - 15)} \\ 9x + 8 \end{array}$$

Completely factor the following polynomials.

47. $6x^3 + 12xy$

$$6x(x^2 + 2y)$$

48. $x^2 - 81$

$$(x-9)(x+9)$$

49. $3x^3 - 27x$

$$3x(x^2 - 9)$$

$$3x(x-3)(x+3)$$

50. $x^2 + 6x + 8$

$$(x+2)(x+4)$$

51. $x^2 - 9x + 14$

$$(x-2)(x-7)$$

52. $x^2 - 6x - 27$

$$(x-9)(x+3)$$

53. $6x^2 - 11x - 2$

$$(6x+1)(x-2)$$

54. $5x^2 + 7x - 6$

$$(5x-3)(x+2)$$

55. $6x^3 - 7x^2 - 10x$

$$x(6x^2 - 7x - 10)$$

$$x(6x+5)(x-2)$$

56. $6xy + 21x + 10y + 35$

$$(6xy + 21x) + (10y + 35)$$

$$3x(2y + 7) + 5(2y + 7)$$

$$(2y+7)(3x+5)$$

57. $6xy + x - 30y - 5$

$$(6xy + x) - (30y + 5)$$

$$x(6y + 1) - 5(6y + 1)$$

$$(6y+1)(x-5)$$

58. $12x^3 - 28x^2 - 3x + 7$

$$(12x^3 - 28x^2) - (3x - 7)$$

$$4x^2(3x - 7) - (3x - 7)$$

$$(3x - 7)(4x^2 - 1)$$

$$(3x-7)(2x-1)(2x+1)$$

59. $x^3 - 125$

$$(x-5)(x^2 + 5x + 25)$$

60. $8x^3 + 27$

$$(2x+3)(4x^2 - 6x + 9)$$

61. $4y^3 - 4$

$$4(y^3 - 1)$$

$$4(y-1)(y^2 + y + 1)$$

Solve the given equations.

62. $(x-2)(x+5) = 0$

$$x-2=0, x+5=0$$

$$2, -5$$

63. $4x^2 - 49 = 0$

$$(2x-7)(2x+7) = 0$$

$$2x-7=0, 2x+7=0$$

$$\frac{7}{2}, -\frac{7}{2}$$

64. $2x^2 + 5x - 10 = x^2 + 2x + 8$

$$x^2 + 3x - 18 = 0$$

$$(x+6)(x-3) = 0$$

$$x+6=0, x-3=0$$

$$-6, 3$$

$$65. 3x^2 - 11x + 14 = (2x+1)(x-2)$$

$$3x^2 - 11x + 14 = 2x^2 - 3x - 2$$

$$x^2 - 8x + 16 = 0$$

$$(x-4)^2 = 0$$

$$x-4=0$$

$$\boxed{4}$$

$$66. (3x+10)(3x-2) = (x+3)(3x+10)$$

$$(3x+10)(3x-2) - (x+3)(3x+10) = 0$$

$$(3x+10)[(3x-2) - (x+3)] = 0$$

$$(3x+10)(2x-5) = 0$$

$$3x+10=0, 2x-5=0$$

$$\boxed{-\frac{10}{3}, \frac{5}{2}}$$

$$67. 4x^3 + 10x^2 - 50x = 0$$

$$2x(2x^2 + 5x - 25) = 0$$

$$2x(2x-5)(x+5) = 0$$

$$2x=0, 2x-5=0, x+5=0$$

$$\boxed{0, \frac{5}{2}, -5}$$

Reduce the following rational expressions to lowest terms.

$$68. \frac{27x^3y}{18x^2y^4}$$

$$\boxed{\frac{3x}{2y^3}}$$

$$69. \frac{x^2 + 4x + 3}{x^2 - 1}$$

$$\frac{(x+1)(x+3)}{(x-1)(x+1)}$$

$$\boxed{\frac{x+3}{x-1}}$$

$$70. \frac{3-x}{3x^2 - 5x - 12}$$

$$\frac{3-x}{(3x+4)(x-3)}$$

$$\frac{-(x-3)}{(3x+4)(x-3)}$$

$$\boxed{-\frac{1}{(3x+4)}}$$

Perform the indicated operations, and reduce the answers to lowest terms.

$$71. \frac{8x^3}{5x^2} \cdot \frac{10x^3}{12x^4}$$

$$\boxed{\frac{4}{3}}$$

$$72. \frac{2x^2 + x}{4x+8} \cdot \frac{2x^2 + 6x}{x^3 + 3x^2}$$

$$\frac{x(2x+1)}{4(x+2)} \cdot \frac{2x(x+3)}{x^2(x+3)}$$

$$\boxed{\frac{2x+1}{2(x+2)}}$$

$$73. \frac{3x^2 - 9xy}{6x^2 - 6y^2} \div \frac{x^3 - 3x^2y}{xy + y^2}$$

$$\frac{3x(x-3y)}{6(x-y)(x+y)} \cdot \frac{y(x+y)}{x^2(x-3y)}$$

$$\boxed{\frac{y}{2x(x-y)}}$$

$$74. \frac{2x^2 + 8x}{2x^2 - 4x - 30} \div \frac{3x^2 - 2x}{3x^2 + 7x - 6}$$

$$\frac{2x(x+4)}{2(x-5)(x+3)} \cdot \frac{(3x-2)(x+3)}{x(3x-2)}$$

$$\boxed{\frac{x+4}{x-5}}$$

$$75. \frac{7x}{9} + \frac{5x}{9}$$

$$\frac{12x}{9}$$

$$\boxed{\frac{4x}{3}}$$

$$76. \frac{6x-5y}{2x-y} - \frac{4x-4y}{2x-y}$$

$$\frac{2x-y}{2x-y}$$

$$\boxed{1}$$

$$77. \frac{x^2 + 3x + 5}{x^2 + 4x + 4} + \frac{x^2 - 3x - 13}{x^2 + 4x + 4}$$

$$\frac{2x^2 - 8}{x^2 + 4x + 4}$$

$$\frac{2(x^2 - 4)}{x^2 + 4x + 4}$$

$$\frac{2(x-2)(x+2)}{(x+2)^2}$$

$$\boxed{\frac{2(x-2)}{x+2}}$$

$$78. \frac{6x+1}{3x-4} - \frac{3x-13}{4-3x}$$

$$\frac{6x+1}{3x-4} + \frac{3x-13}{3x-4}$$

$$\frac{9x-12}{3x-4}$$

$$\frac{3(3x-4)}{3x-4}$$

$$\boxed{3}$$

$$79. \frac{11x+2}{2x-4} - \frac{2x+15}{2x-4} - \frac{3x-1}{2x-4}$$

$$\frac{6x-12}{2x-4}$$

$$\frac{6(x-2)}{2(x-2)}$$

$$\boxed{3}$$

Given the following linear equations, complete the table of solution pairs.

80. $6x - 5y = 10$

x	y
0	$\boxed{-2}$
$\boxed{\frac{5}{3}}$	0
$\boxed{\frac{10}{3}}$	2

81. $3x - 4y + 9 = 0$

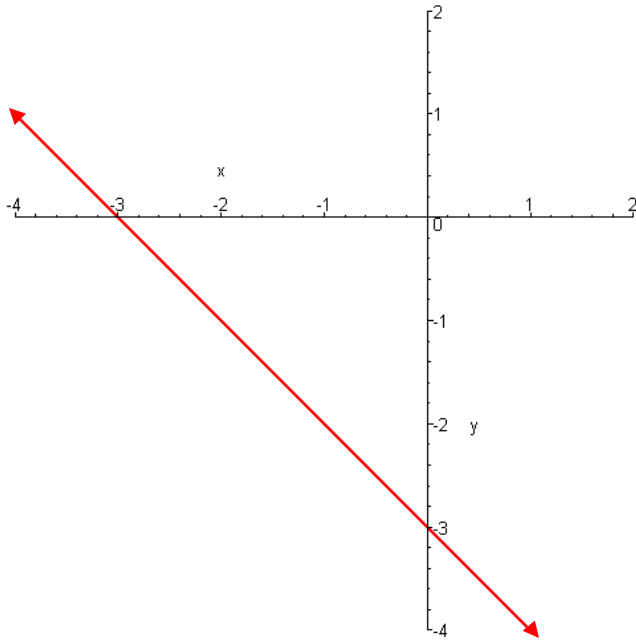
x	y
-3	$\boxed{0}$
$\boxed{-3}$	0
5	$\boxed{6}$

82. $x + y = 0$

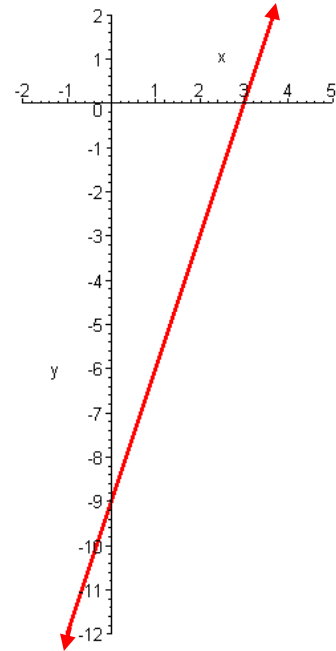
x	y
1	$\boxed{-1}$
$\boxed{-2}$	2
-5	$\boxed{5}$

Graph the solutions of the following linear equations. Indicate the intercept(s).

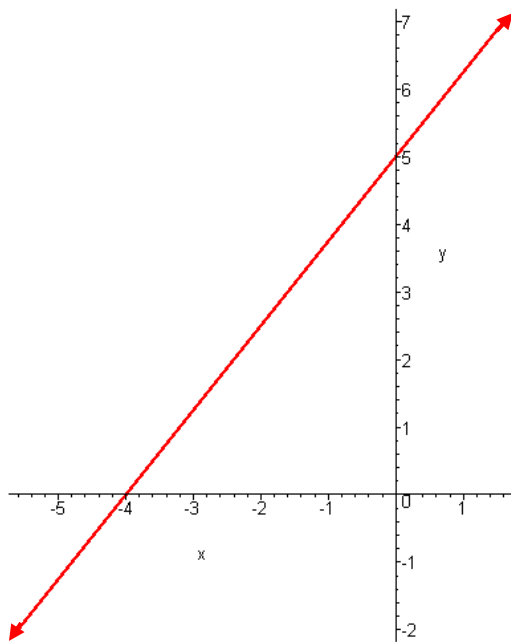
83. $x + y = -3$



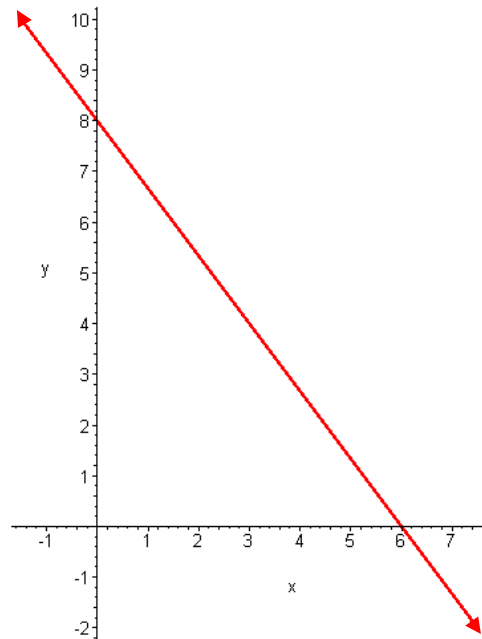
84. $3x - y = 9$



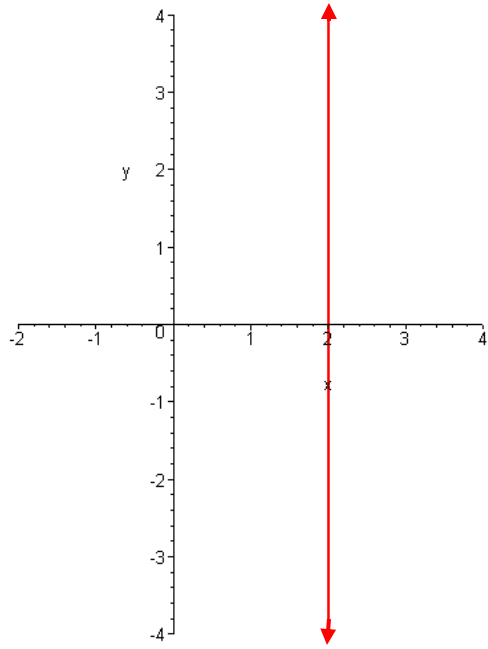
85. $5x - 4y = -20$



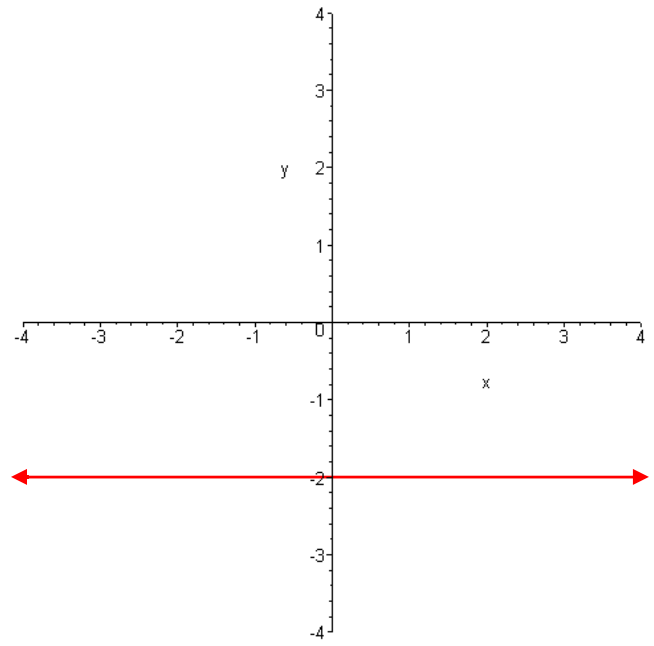
86. $4x + 3y = 24$



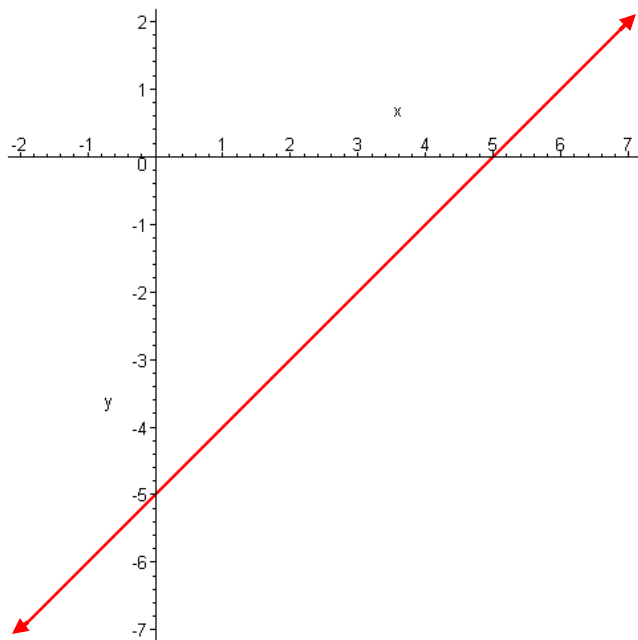
87. $x = 2$



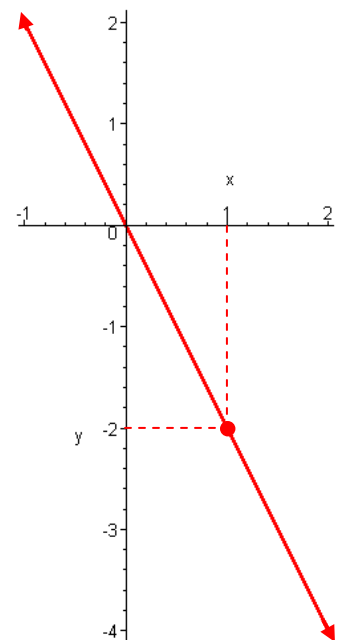
88. $3y + 6 = 0$



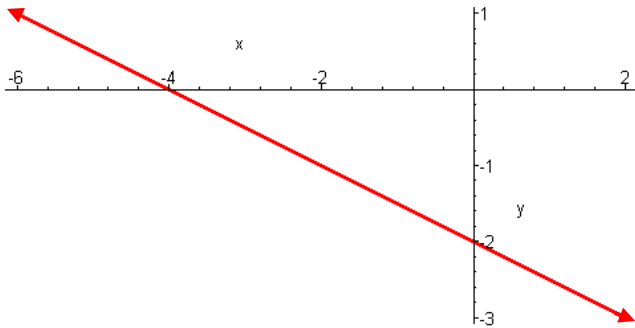
89. $-x + y = -5$



90. $2x + y = 0$



91. $-x - 2y = 4$



Determine the solutions of the following linear systems of equations. If there are no solutions, write *inconsistent*. If there are infinitely many solutions, write *dependent*.

92. $3x - 7y = 8$
 $x = -2$

$$-6 - 7y = 8$$

$$-7y = 14$$

$$\boxed{y = -2}$$

$$\boxed{x = -2}$$

93. $x = 3y + 1$
 $2x - 5y = 3$

$$2(3y + 1) - 5y = 3$$

$$6y + 2 - 5y = 3$$

$$y + 2 = 3$$

$$\boxed{y = 1}$$

$$\boxed{x = 4}$$

94. $3x + 8y = 1$
 $x + y = 2$

$$3(2 - y) + 8y = 1$$

$$6 - 3y + 8y = 1$$

$$5y + 6 = 1$$

$$5y = -5$$

$$\boxed{y = -1}$$

$$\boxed{x = 3}$$

95. $2x - 6y = 4$
 $-x + 3y = -2$

$$2x - 6y = 4$$

$$+ 2(-x + 3y = -2)$$

$$0 = 0$$

infinitely many solutions

$\boxed{\text{dependent}}$

96. $6x - y = -1$
 $3x + 2y = 12$

$$2(6x - y = -1)$$

$$+(3x + 2y = 12)$$

$$15x = 10$$

$$\boxed{x = \frac{2}{3}}$$

$$\boxed{y = 5}$$

97. $5x + y = -14$
 $2x - y = -7$

$$5x + y = -14$$

$$+(2x - y = -7)$$

$$7x = -21$$

$$\boxed{x = -3}$$

$$\boxed{y = 1}$$

$$98. \begin{aligned} -3x + 7y &= -7 \\ 3x + 2y &= 25 \end{aligned}$$

$$\begin{aligned} -3x + 7y &= -7 \\ + (3x + 2y &= 25) \end{aligned}$$

$$9y = 18$$

$$\boxed{\begin{aligned} y &= 2 \\ x &= 7 \end{aligned}}$$

$$99. \begin{aligned} 6x - 4y &= 10 \\ -3x + 2y &= -4 \end{aligned}$$

$$\begin{aligned} 6x - 4y &= 10 \\ + 2(-3x + 2y &= -4) \end{aligned}$$

$$0 = 2$$

no solution

inconsistent

$$100. \begin{aligned} 11x + 10y &= 38 \\ 4x + 11y &= 31 \end{aligned}$$

$$\begin{aligned} -4(11x + 10y &= 38) \\ + 11(4x + 11y &= 31) \end{aligned}$$

$$81y = 189$$

$$\boxed{\begin{aligned} y &= \frac{7}{3} \\ x &= \frac{4}{3} \end{aligned}}$$

Simplify the following radicals, if possible. Assume all variables represent positive numbers.

$$101. \sqrt{49}$$

$$\boxed{7}$$

$$102. \sqrt{\frac{100}{9}}$$

$$\boxed{\frac{10}{3}}$$

$$103. \sqrt[3]{\frac{-27}{125}}$$

$$\boxed{-\frac{3}{5}}$$

$$104. \sqrt{63}$$

$$\boxed{3\sqrt{7}}$$

$$105. \sqrt{13x^{10}}$$

$$\boxed{x^5\sqrt{13}}$$

$$106. \sqrt{80x^4y^7}$$

$$\boxed{4x^2y^3\sqrt{5y}}$$

$$107. \sqrt{275x^{11}y^{25}}$$

$$\boxed{5x^5y^{12}\sqrt{11xy}}$$

$$108. \sqrt{\frac{11}{144}}$$

$$\boxed{\frac{\sqrt{11}}{12}}$$

$$109. \sqrt{\frac{99x^{16}}{16}}$$

$$\boxed{\frac{3x^8\sqrt{11}}{4}}$$

Solve the following equations.

$$110. \frac{3x}{5} - \frac{x}{2} = \frac{7}{10}$$

$$6x - 5x = 7$$

$$x = 7$$

$$\boxed{7}$$

$$111. \frac{x+4}{x-1} = \frac{6}{x-3}$$

$$x^2 + x - 12 = 6x - 6$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$\boxed{6, -1}$$

$$112. \frac{x}{x+1} - \frac{5}{x-3} = \frac{-20}{x^2 - 2x - 3}$$

$$x(x-3) - 5(x+1) = -20$$

$$x^2 - 8x - 5 = -20$$

$$x^2 - 8x + 15 = 0$$

$$(x-3)(x-5) = 0$$

$$\boxed{5}$$