
Chapter 2

Solving Equations and Inequalities

Section 2.1: Linear Equations

- Solving an Equation
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Solving an Equation

Solving an Equation

To solve an equation in the variable x using the **algebraic method** is to use the rules of algebra to isolate the unknown x on one side of the equation.

To solve an equation in the variable x using the **graphical method** is to move all terms to one side of the equation and set those terms equal to y . Sketch the graph to find the values of x where $y = 0$.

Algebraic Method: Solve the equation $-x - 1 = 2x + 2$ by the algebraic method.

Isolate x on one side of the equation.

$$\begin{aligned} -x - 1 &= 2x + 2 \\ -3x - 1 &= 2 \\ -3x &= 3 \\ x &= -1 \end{aligned}$$

The solution is $x = -1$.

CHAPTER 2 Solving Equations and Inequalities

Graphical Method: Solve the equation $-x - 1 = 2x + 2$ by the graphical method.

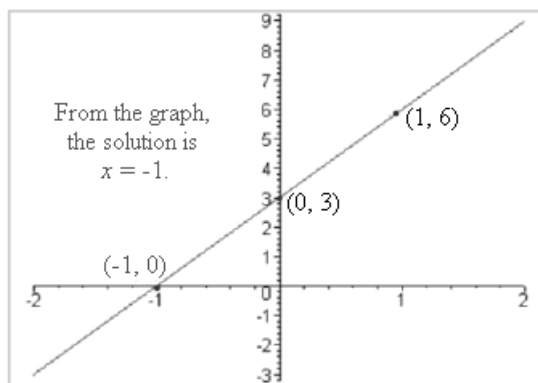
Move all terms to one side of the equation.

$$-x - 1 = 2x + 2$$

$$-1 = 3x + 2$$

$$0 = 3x + 3$$

Set $y = 3x + 3$ and graph. The graph is a line with slope 3 and y -intercept 3. Use the y -intercept 3 to plot the point $(0, 3)$. Use the slope $3 = \frac{3}{1}$ to move 3 units up and 1 unit to the right to locate another point on the graph. Draw a line through the two points and extend the line until it crosses the x -axis.

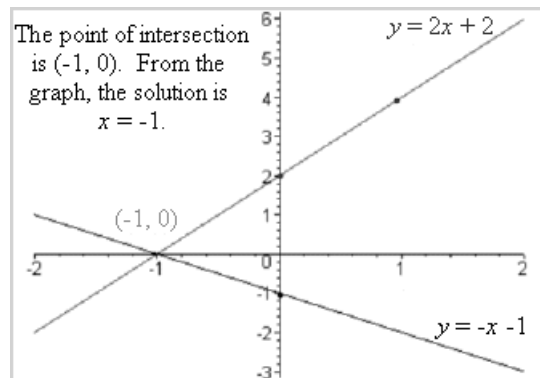


Graphical Method (Another Approach): Solve the equation $-x - 1 = 2x + 2$ by the graphical method.

Instead of moving all terms to one side of the equation, graph two equations:

$$y = -x - 1 \quad \text{and} \quad y = 2x + 2$$

The solution is the x -coordinate of the point of intersection of the two lines.



Example Problem: Solve the equation $x^2 - 4 = 0$ both algebraically and graphically.

Solution:

Algebraic Method: Isolate x on one side of the equation.

$$x^2 - 4 = 0$$

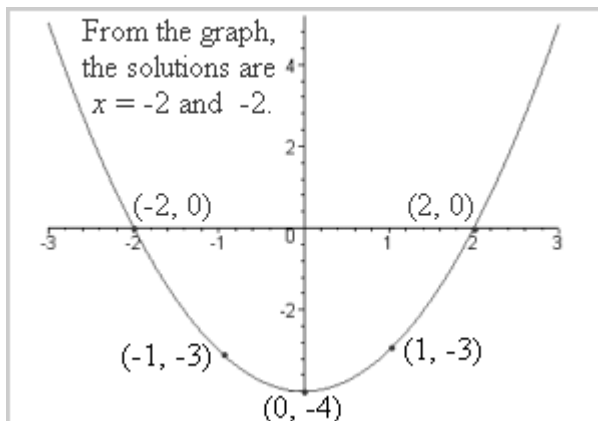
$$x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$x = \pm 2$$

The solutions are $x = 2$ and $x = -2$.

Graphical Method: Set $y = x^2 - 4$. The graph is a parabola. Make a table of values and sketch the graph.



Additional Example 1:

Solve the equation $6x + 3 = 4x + 33$.

Solution:

$$6x + 3 = 4x + 33$$

$$6x + 3 - 4x = 4x + 33 - 4x$$

$$2x + 3 = 33$$

$$2x + 3 - 3 = 33 - 3$$

$$2x = 30$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{30}{2}$$

$$x = 15$$

Additional Example 2:Solve the equation $2(x-3)+7=-4(x+1)+3$.**Solution:**

$$2(x-3)+7=-4(x+1)+3$$

$$2x-6+7=-4x-4+3$$

$$2x+1=-4x-1$$

$$2x+1+4x=-4x-1+4x$$

$$6x+1=-1$$

$$6x+1-1=-1-1$$

$$6x=-2$$

$$\cancel{6}x = \frac{-2}{\cancel{6}}$$

$$x = -\frac{1}{3}$$

Additional Example 3:Solve the equation $2x + \frac{x}{12} + \frac{x-3}{6} = x$.**Solution:**

$$2x + \frac{x}{12} + \frac{x-3}{6} = x$$

We first multiply both sides of the equation by 12 to clear the equation of fractions. Then solve as usual.

$$12(2x) + \frac{12x}{12} + \frac{12(x-3)}{6} = 12x$$

$$24x + x + 2(x-3) = 12x$$

$$24x + x + 2x - 6 = 12x$$

$$27x - 6 = 12x$$

$$27x - 6 - 12x = 12x - 12x$$

$$15x - 6 = 0$$

$$15x - 6 + 6 = 0 + 6$$

$$15x = 6$$

$$\cancel{15}x = \frac{6}{\cancel{15}}$$

$$x = \frac{6}{15}$$

$$x = \frac{2}{5}$$

Additional Example 4:

Solve the equation $\frac{1}{2}x + 1 = 3$ graphically.

Solution:

Move all terms to one side of the equation. To do this, subtract 3 from both sides.

$$\frac{1}{2}x + 1 = 3$$

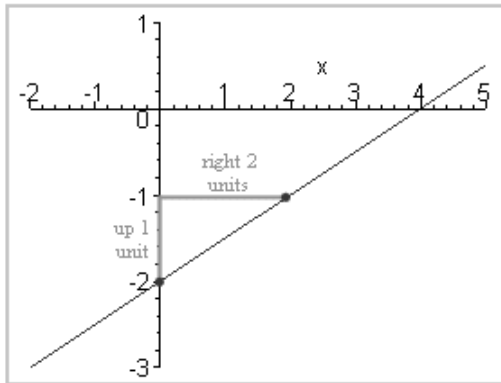
$$\frac{1}{2}x - 2 = 0$$

Set the LHS equal to y .

$$y = \frac{1}{2}x - 2$$

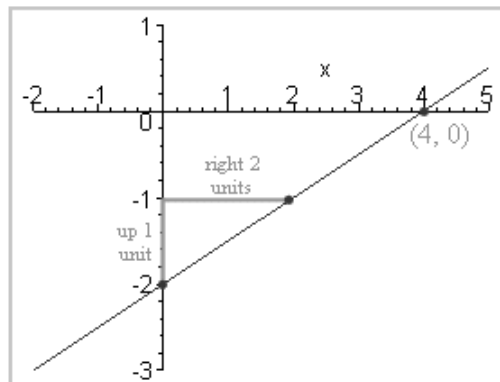
Sketch the graph of the equation $y = \frac{1}{2}x - 2$. The graph is a line with slope

$\frac{1}{2}$ and y -intercept -2 . Extend the line until it crosses the x -axis.



Look on the graph to find the value of x where $y = 0$.

The solution is $x = 4$.



Exercise Set 2.1: Linear Equations

Solve the following linear equations algebraically.

1. $-3x + 7 = 13$

2. $5x - 11 = 6$

3. $2x + 3 = 4x - 7$

4. $5x + 2 = -4x - 6$

5. $3(x + 2) + 9 = -5(x - 8) - 3$

6. $-4(x + 3) - 5 = 2(x - 4) + 3$

7. $3(2 - 5x) = -4(7x - 3)$

8. $7 + 2(3 - 8x) = 4 - 6(1 + 5x)$

9. $\frac{x}{5} = -7$

10. $\frac{x}{3} = 10$

11. $\frac{2}{5}x - 1 = 7$

12. $-\frac{3}{4}x - 7 = 2$

13. $\frac{5}{3}(x - 7) = \frac{2}{5}x + 1$

14. $\frac{4}{9}x - 12 = -\frac{1}{6}(x - 12) - 3$

15. $2 + \frac{2x}{3} - \frac{x + 5}{7} = 3x$

16. $x + \frac{x + 7}{8} + \frac{5x}{6} = \frac{-1}{12}$

19. $\frac{4}{x - 5} - \frac{1}{3} = \frac{-8}{3x - 15}$

20. $\frac{7}{x + 2} + \frac{4}{3x + 6} = -\frac{5}{3}$

Solve each of the following linear equations graphically, and then check your answer algebraically.

21. $2x - 5 = 3$

22. $-3x - 1 = 5$

23. $\frac{2}{3}x - 3 = 1$

24. $-\frac{3}{5}x - 1 = -4$

25. $2x + 3 = \frac{3}{4}x - 2$

26. $\frac{2}{3}x - 5 = -\frac{1}{2}x + 2$

Solve the following nonlinear equations algebraically.

(Note: Even though these equations are not linear, some steps in the solution will contain a linear equation.)

17. $\frac{2}{5x} + \frac{7}{4x} = -3$

18. $-\frac{7}{6x} + \frac{5}{4x} = 2$