



# Grade 4 Concept Maps

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## Concept Map for *Mathematical Thinking at Grade 4*

In this unit, students are familiarized with the mathematics content and the approaches of the *Investigations* curriculum. The unit can be used to assess the strengths and needs of the students at the beginning of the year and can provide an introduction to some of the key mathematical concepts and topics that will be covered in more depth later in the year

**INVESTIGATION 1: *How Many Hundreds?*** – Students learn to group items for more efficient counting; reorder numbers for more efficient mental arithmetic; explore materials they will be using throughout the year; estimate how many hundreds are in the total of a group of three-digit numbers; communicate their mathematical thinking through spoken and written language

**Session 1: Getting Started with Interlocking Cubes**

Students estimate and then count how many cubes were used to build an object; write about their mathematical thinking

**Sessions 2, 3: How Many Hundreds?**

The class works on estimating skills; keeping track of data; adding three-digit numbers

**Session 4: Close to 100**

Students play *Close to 100*, a game that involves arranging digits to make pairs of numbers that equal or come close to 100

**INVESTIGATION 2: *How Many Dollars?*** – Students group coins for efficient counting; recognize values of U.S. coins; recognize use of the decimal point on the calculator and when working with money

**Sessions 1, 2: How Much Money?**

Students recognize the values of U.S. coins; organize and keep track of the money they are counting; find amounts in coins with a total of one or more dollars

**Sessions 3, 4: Number Sense and Coins**

Students make Choice Time decisions; make numbers whose sum is close to 100; add coins to make dollar amounts

**INVESTIGATION 3: *Using Number Patterns*** – Students focus on using known answers to find unknown ones; subtract on a 300 chart and with a calculator; add and subtract multiples of ten

**Sessions 1, 2: The 300 Chart**

Students write numbers from 1 to 300; count by 10's and multiples of 10; find differences between numbers; use a calculator to subtract

**Session 3: Related Problem Sets**

Students work on a set of closely related problems involving addition, subtraction, and money; consult with one another about their work and their strategies

**Sessions 4, 5: Addition and Subtraction Strategies (Assessment)**

Students demonstrate their facility with mental addition and subtraction; add and subtract multiples of 10

**INVESTIGATION 4: *Making Geometric Patterns*** – Students distinguish between geometric patterns and random designs; distinguish between mirror symmetry and rotational symmetry; write about their designs

**Session 1: Patterns with Mirror Symmetry**

Students make patterns with mirror symmetry; copy patterns onto paper; work cooperatively with a partner

**Session 2: Patterns with Rotational Symmetry**

Students make patterns with rotational symmetry; copy patterns onto paper; think about other kinds of patterns

**Sessions 3, 4: Patterns and Nonpatterns**

Students recognize patterns; describe a visual design orally and in writing; build a design following oral instructions

**Sessions 5, 6: Symmetrical Geoboard Patterns**

Students use geoboards to make symmetrical patterns; copy patterns onto paper; find symmetry in other students' patterns

## Concept Map for *Arrays and Shares*

Students work on skip counting, building arrays, dividing quantities into equal shares, and solving word problems. They develop an understanding of how multiplication and division are related; gain fluency with multiplication and division pairs; solve problems by breaking them into more manageable components; develop strategies based on what they know about numbers

**INVESTIGATION 1: *Multiples on the 100 Chart*** – Students use skip counting as a model for multiplication; see multiplication as an accumulation of groups of a number; look for multiplication patterns; interpret standard multiplication and division notation; use known multiplication relationships to solve more complicated problems

**Sessions 1, 2: Multiples on the 100 Chart**  
Students practice skip counting and look for the multiplication patterns of numbers 2 to 12 on the 100 chart

**Session 3: Skip Counting and Multiplying**  
Students represent multiplication; interpret multiplication and division notation; solve related multiplication problems

**INVESTIGATION 2: *Arrays*** – Students use an array as a model for multiplication; become more familiar with multiplication pairs; recognize prime numbers; use a variety of notations for multiplication and division; understand how division notation represents a variety of division situations (including sharing and partitioning); determine what to do with “leftovers” in division

**Session 1: Things That Come in Arrays**  
Students use arrays as models for multiplication; use skip counting to count arrays; relate multiplication notation to arrays

**Sessions 2, 3: Making Arrangements**  
Students find factors of numbers; recognize prime numbers; use an array as a model for multiplication

**Session 4: Preparing a Set of Arrays**  
Students again use arrays as models for multiplication, in the process learning more about multiplication pairs

**Sessions 5, 6: Array Games**  
Students increase their familiarity with multiplication and division pairs through arrays and skip counting

**Sessions 7, 8: Looking at Division**  
Students recognize division situations; use division notation; solve division problems; express remainders using story-problem contexts

**INVESTIGATION 3: *Multiplication and Division with Two-Digit Numbers*** – Students become fluent in basic multiplication relationships; partition numbers to multiply them more easily; recognize multiplication and division situations and represent each situation using a mathematical statement; use patterns when multiplying by multiples of 10

**Session 1: Multiplication Clusters**  
Students use multiplication relationships and break down large problems into smaller parts

**Sessions 2, 3, 4: Multiplication & Division Choices**  
Students recognize and solve multiplication/division problems; use multiplication and division notation

**Session 5: Problems That Look Hard But Aren't**  
Students apply their knowledge of multiplication relationships to solving two-digit problems; use what they know about multiplication in order to discuss and solve harder problems

## Concept Map for *Seeing Solids and Silhouettes*

Students develop spatial-visualization skills (an often-neglected part of geometry) as they explore ways to pictorially represent solid shapes; investigate silhouettes projected by geometric solids; explore what objects look like from different perspectives; work on ways to effectively communicate their ideas about three-dimensional objects

**INVESTIGATION 1: *Making and Visualizing Cube Buildings*** – Students develop the concepts and language needed to reflect on and discuss spatial relationships in 3-D environments; work on understanding standard drawings of 3-D cube configurations; explore spatial relationships among components of 3-D figures; develop visualization skills; begin to think about problems related to volume

**Session 1: Building with Cubes**

Students understand standard drawings of 3-D cube configurations; explore the spatial relationships among the components of cube configurations

**Session 2: Making Mental Pictures**

Students develop visualization skills and the language needed to reflect on and communicate their ideas about spatial relationships

**INVESTIGATION 2: *Exploring Geometric Silhouettes*** – Students see how 3-D geometric solids project shadows having 2-D shapes; learn more about geometric perspective; learn to visualize objects from different perspectives; integrate different views of an object to form mental models of the whole

**Sessions 1, 2: Silhouettes of Geometric Solids (Excursion)**

Students increase their understanding of geometric perspective; learn to visualize what objects look like from different perspectives

**Sessions 3, 4: Silhouettes of Cube Buildings**

Students continue to work on geometric perspective and spatial visualization; integrate different views of an object to form a mental image of the whole object

**Session 5: Different Views of a City (Excursion)**

Students use a “map” of a cube city to try to understand geometric perspective and visualize what objects look like from different perspectives

**INVESTIGATION 3: *“How-To” Instructions for Cube Buildings*** – Students interpret different types of instructions for building with cubes; evaluate the effectiveness of different forms of “how-to” instructions; develop visualization skills; integrate information gathered from different views (or from discussion) to form one coherent mental model of a cube building

**Session 1: Writing “How-To” Instructions**

Students communicate in writing about three-dimensional objects; evaluate the effectiveness of their own communication; develop visualization skills

**Sessions 2, 3: Which Instructions Are Best?**

Students continue to interpret different types of instructions; evaluate the effectiveness of different forms of “how-to” instructions

**INVESTIGATION 4: *The Cube Toy Project*** – Students work in centers creating their own interlocking-cube “toys” and write instructions that can enable other people to build those toys. The focus is on effective communication skills when describing three-dimensional objects, and on revising written instructions as necessary

**Sessions 1, 2, 3, 4: Making Plans for a Cube Toy**

Students make a cube toy and write clear and complete instructions for making that toy

## Concept Map for *Landmarks in the Thousands*

Students explore the structure of our number system through activities involving hundreds and thousands; explore factors of 100 and 1000 (important landmarks in our number system); use this experience to work with multiples of 100 and 1000; use these landmarks when solving addition and subtraction problems in the hundreds; develop a sense of the magnitude of 100, 1000, and 10,000

**INVESTIGATION 1: *Working with 100*** – Students find and count by factors of 100; recognize factor pairs; use landmarks to find differences between numbers under 100; make conjectures about factors of 100

**Session 1: Ways to Count to 100**

Students find and count by factors of 100; develop conjectures about how to tell which numbers are factors of 100

**Session 2: 100 in a Box**

Students use factors of 100 to build rectangular solids using a total of 100 cubes; keep track of and record their findings

**Session 3: Moving Around on the 100 Chart**

Students find the difference between any 2-digit number and 100; cite landmarks on the 100 chart; add to and subtract 10 from 2-digit numbers

**INVESTIGATION 2: *Exploring Multiples of 100*** – Students use their knowledge of factors of 100 to explore multiples of 100; relate knowledge of factors to division situations and to standard division notation; add multiples of 10 to and subtract multiples of 10 from numbers in the hundreds; solve addition and subtraction problems by reasoning from known relationships; communicate strategies

**Session 1: Factors of 100, 200, and 300**

Students use their knowledge about the factors of 100 to find factor pairs for multiples of 100; count by factors of 100 to larger multiples of 100; interpret standard division notation as a relationship between a factor and its multiple; look for patterns in factors of 300

**Sessions 2, 3, 4: Using Landmarks to Add and Subtract**

Students add and subtract numbers in the hundreds; use landmarks and skip counting to solve problems; find factors of 200 and 300; add and subtract multiples of 10 from numbers on the 300 chart

**Session 5: Solving Problems in the Hundreds**

Students use their knowledge of the factors of 100, 200, and 300 to solve problems; communicate about their strategies with words, pictures, and numbers

**INVESTIGATION 3: *How Much Is 1000?*** – Students read and write numbers to 1000; locate numbers in sequence to 1000; get a sense of the magnitude of multiples of 100 up to 1000; identify and use important landmarks up to 1000; develop strategies for adding and subtracting numbers in the hundreds; estimate quantities to 1000

**Session 1: Numbers to 1000**

Students read, write, and sequence numbers to 1000

**Session 2: Moving Around in the 1000 Book**

Students use what they know about factors and multiples to identify factors of 1000; count by 100's, 50's, and 25's to 1000; develop strategies for subtracting numbers in the hundreds

**Sessions 3, 4, 5: Estimating, Adding, and Subtracting to 1000**

Students continue their work on adding and subtracting numbers in the hundreds; estimate quantities of objects in the hundreds

**INVESTIGATION 4: *Making a 10,000 Chart*** – Students read, write, and sequence numbers in the thousands; get a sense of the magnitude of, and understand the structure of, 10,000; add multiples of 100 to and subtract multiples of 100 from numbers in the thousands

**Sessions 1, 2, 3: 10,000 Squares on the Wall**

Students read, write, and understand the sequence of numbers in the thousands; and multiples of 100 to and subtract multiples of 100 from numbers in the thousands

## Concept Map for *Different Shapes, Equal Pieces*

Students explore fractions by dividing square areas into halves, fourths, and eighths and rectangular areas into thirds, sixths, and twelfths. They also work on ordering fractions, including those greater than 1, and on identifying fraction equivalents

**INVESTIGATION 1: *Parts of Squares: Halves, Fourths, and Eighths*** – Students begin to understand that equal fractions of a whole have the same area, and that equal shapes are not necessarily congruent, as they become familiar with relationships among halves, fourths, and eighths

<p style="text-align: center;"><b>Session 1:</b> <b>Finding Halves of Crazy Cakes</b></p> <p>Students focus on the idea that equal fractions of a whole each have the same area; find halves of different shapes</p>	<p style="text-align: center;"><b>Sessions 2, 3, 4:</b> <b>Halves, Fourths, and Eighths with Geoboards</b></p> <p>Students explore relationships among halves, fourths, and eighths; understand that equal parts of a whole must be exactly equal in size but are not necessarily congruent; justify their division of an area into equal parts</p>	<p style="text-align: center;"><b>Session 5:</b> <b>Combining Fractions in a Design</b></p> <p>Students construct ways of dividing a whole into combinations of halves, fourths, and eighths; write equations that reflect pictures with halves, fourths, and eighths</p>
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**INVESTIGATION 2: *Parts of Rectangles: Thirds, Sixths, and Twelfths*** – Students become familiar with relationships among thirds, sixths, and twelfths; use different combinations to make a whole; compare fractions that each have “one piece missing”; work with fractions that have numerators greater than 1

<p style="text-align: center;"><b>Sessions 1, 2:</b> <b>Thirds, Sixths, and Twelfths</b></p> <p>Students use a larger whole to explore size and equivalence relationships among fractions based on one-third; justify their divisions of rectangles into thirds, sixths, and twelfths</p>	<p style="text-align: center;"><b>Session 3:</b> <b>More Fraction Designs</b></p> <p>Students visualize the relationships among <math>1/2</math>, <math>1/3</math>, <math>1/4</math>, <math>1/6</math>, <math>1/8</math>, and <math>1/12</math>; write equations that reflect the relationships among these fractions</p>	<p style="text-align: center;"><b>Session 4:</b> <b>Working with <math>2/3</math>, <math>3/4</math>, <math>5/6</math>, and <math>7/8</math></b></p> <p>Students use measurement to divide a square into fractional parts; compare fractions that each have “one piece missing”; work with fractions that have numerators greater than 1</p>
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**INVESTIGATION 3: *Ordering Fractions*** – Students compare fractions to landmarks 0,  $1/2$ , 1, and 2; use numerical reasoning and area to order fractions; use the size of the numerator to compare fractions having like denominators; use the size of the denominator to compare fractions having equal numerators; identify equivalent fractions

<p style="text-align: center;"><b>Sessions 1, 2:</b> <b>Making Fraction Cards</b></p> <p>Students make fraction cards; develop visual representations of fractions; recognize equivalence of fractions such as <math>1/2</math> and <math>2/4</math> both visually and numerically</p>	<p style="text-align: center;"><b>Session 3:</b> <b>Ordering Fractions with Respect to Landmarks</b></p> <p>Students compare fractions with landmarks 0, <math>1/2</math>, 1, and 2; discriminate among fractions less than 1 and greater than 1</p>	<p style="text-align: center;"><b>Sessions 4, 5:</b> <b>Making a Fraction Number Line</b></p> <p>Students compare relative sizes of common fractions; compare fractions with <math>1/2</math> and 1; compare fractions that are each missing one piece of a whole</p>
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## Concept Map for *The Shape of the Data*

Students are provided with some of the tools to record, represent, and analyze simple data sets about familiar situations. They define issues and questions about the data they collect; organize data and make presentation graphs; look at the shape of the data (patterns and special features) as they identify places where there is a concentration of data (clumps) or where there are no data (holes); describe what seems to be typical for a set of data

**INVESTIGATION 1: *Introduction to Data Analysis*** – Students make quick sketches of the data to use as working tools during the analysis process; describe the shape of the data, moving from noticing individual features of the data to describing the overall shape of the distribution; define ways data will be collected; summarize what is typical of a set of data

**Session 1: How Many Raisins in a Box?**  
 Students count the raisins in a sample of small boxes of raisins; organize, record, graph, and examine the shape of the data they collect

**Sessions 2, 3: How Many People in a Family?**  
 Students work to define the way data will be collected; collect the data; sketch and describe their data; summarize what is typical of the data

**INVESTIGATION 2: *Landmarks in the Data*** – Students describe the shape of the data; summarize what is typical of the data; invent ways to compare two sets of data; use linear measurement; visualize and estimate lengths and heights; use the median to describe a set of data and to compare one data set to another; find the median in a data set arranged in numerical order or grouped by frequency

**Session 1: How Tall Are Fourth Graders?**  
 Students measure accurately; record and describe the data; identify what is typical of the data set; propose methods for solving a problem

**Sessions 2, 3: Fourth and First Graders: How Much Taller?**  
 Students continue to practice measuring accurately; compare two sets of data; prepare presentation graphs and reports

**Session 4: Looking at Mystery Data**  
 Students describe mystery data and construct theories based upon the data; examine the relationship between grouped and ungrouped data; visualize and estimate lengths and heights

**Session 5: Finding the Median**  
 Students work with the idea of median; find the median in different kinds of data sets; use the median to compare two data sets

**Sessions 6, 7: Using Landmarks in Data**  
 After examining another set of mystery data, students organize the data; describe the landmarks (such as the median, outliers, and clusters), using the median to compare data sets

**INVESTIGATION 3: *A Data Project: Investigating Sleep*** – Students undertake a complete data-analysis project (from defining a question to publishing the results); choose and refine a research question; view the data in several different ways (use quick sketches and other representations to organize and display the data)

**Sessions 1, 2: What Do We Want to Find Out?**  
 The class plans an extended investigation; chooses and refines an investigation question; collects, records, organizes, describes, and interprets data; refines and improves its own research strategies

**Sessions 3, 4, 5: The Research Team at Work**  
 Students carry out their extended investigation; represent the data in several ways; prepare a publication that includes a statement of the research question, a presentation graph, a written description of the data, and an interpretation of the findings

## Concept Map for *Money, Miles, and Large Numbers*

Students add and subtract decimal numbers and numbers in the hundreds and thousands; work with combining situations in which they determine how several numbers combine to make a given number, and comparing situations that involve finding the difference between two quantities. Emphasis throughout the unit is placed on students developing their own sound strategies for adding and subtracting, especially with money and larger numbers, as they move toward computational fluency

**INVESTIGATION 1: *Everyday Uses of Money*** – Students estimate sums and total amounts of money; explore number relationships in the context of money and develop strategies for combining monetary amounts; use landmarks .10, 10, 100, and \$1.00 to compare and to find the difference between two quantities; use standard addition and subtraction notation to record combining and comparing situations; interpret decimals on the calculator as amounts of money

**Sessions 1, 2: Groceries, Lunch, and Book Orders**  
Students estimate totals for amounts of money; add money

**Session 3: Making a Dollar**  
Students find combinations to total \$1.00 or 100

**Sessions 4, 5: Making Sense (Cents) of Money on the Calculator**  
Students interpret calculator displays as amounts of money; add money on the calculator

**Session 6: Making Change**  
Students calculate change and count up to make change

**Sessions 7, 8: Shopping Smart**  
Students estimate and calculate total amounts; make change; write number sentences to represent problems

**INVESTIGATION 2: *How Far? Measuring in Miles and Tenths*** – Students estimate local distances in miles and tenths of miles; develop a sense of how long 1 mile and 1/10 of a mile are; compare and combine decimal numbers; find differences between decimal numbers; see the relationships of decimal parts to the whole; measure distances on maps using a scale; become familiar with common decimal and fraction equivalents; estimate and calculate sums that include decimal portions

**Sessions 1, 2:  
Miles and Tenths of a Mile**  
Students measure using miles, tenths of miles, and quarters of miles; become familiar with common fraction/decimal equivalents; visualize distances such as 1/10 of a mile; add numbers that include whole numbers, tenths, and quarters

**Session 3:  
How Far Is 1/10 of a Mile? (Excursion)**  
Students determine and visualize how long 1/10 of a mile is; invent ways to measure large distances

**Session 4:  
A Tour of Our Town**  
Students combine and compare decimal amounts; determine distances using a scale of miles

**INVESTIGATION 3: *Calculating Longer Distances*** – Students measure distances on maps using a scale; compare and combine numbers in the hundreds and thousands; use standard addition and subtraction notation to record combining and comparing problems

**Session 1:  
Close to 1000**  
Students play *Close to 1000*, a game in which they find combinations of numbers that are close to 1000

**Sessions 2, 3, 4:  
A Trip Around the United States**  
Students use a map scale; measure distances on a map of the United States; combine numbers in the hundreds and thousands

## Concept Map for *Changes Over Time* (Page 1 of 2)

Students explore the ways that things change over time and look at ways to represent these changes; make representations that show how one thing changes over time; make charts and line graphs to show changing speed, changing heights of plants over several weeks, and changing population; interpret the shapes of the curves in line graphs

**UNIT PREPARATION: *Growing Plants to Graph*** – Students prepare for this unit by planting beans and by following the growth of their plants for four weeks. The general schedule is as follows:

**Prep. Session 1: Sprouting the Seeds**

Students share their ideas about how plants grow and then prepare their seeds for sprouting

**Prep. Session 2: Planting the Seeds**

Students plant their sprouted seeds in soil and begin to collect population data from their homes

**Prep. Session 3: Measuring the Plants**

Students measure the growth of the seedlings to the nearest half-centimeter; keep track of daily measurements on a chart

**INVESTIGATION 1: *Graphing Population Changes*** – Students make graphs that show change over time; invent representations of data; interpret different kinds of graphs; develop a scale that includes all the data; decide how to group the data; establish conventions for consistency; understand how changes and total are related; develop strategies for solving and writing missing-information problems

**Sessions 1, 2:  
Graphing Home Population Over Time**

Students make graphs that show changes over time; invent representations; interpret different kinds of graphs

**Sessions 3, 4:  
Making Group Graphs**

Students develop a scale that includes all the data; decide how to group data; establish conventions for consistency

**Sessions 5, 6:  
Ins and Outs Number Problems**

Students learn how changes and total are related; develop strategies for solving missing-information problems; write such problems themselves

**INVESTIGATION 2: *Ways to Show Changes Over Time*** – Students examine real situations and events that show change; interpret representations that show change; make representations that show change; distinguish between representations of something that can change and representations that show changes

**Sessions 1, 2:  
Showing Change Over Time**

Students' work focuses on representations of change and how to interpret and create them

## Concept Map for *Changes Over Time* (Page 2 of 2)

**INVESTIGATION 3: *Telling Stories from Line Graphs*** – Students attend to the shapes of graphs; use curves to communicate information; develop an understanding of the difference between continuous and discrete changes; describe growth with a line graph; compare graphs of slow growth and fast growth; develop a scale that includes all the data; make, interpret, and compare line graphs; integrate quantitative, qualitative, and graphical descriptions of the same data; tell a story from a curve; make and interpret different graphical shapes

**Sessions 1, 2: Making Graph Sketches**

Students attend to the shapes of graphs; use curves to communicate information

**Session 3: Graphing and Predicting Plant Growth**

Students develop an understanding of the difference between continuous and discrete changes; describe growth with a line graph; compare graphs of slow growth and fast growth

**Session 4: Using Line Graphs to Compare Growth**

Students develop a scale that includes all the data; make, interpret, and compare line graphs

**Session 5:  
Graphs, Stories, and Number Sequences**

Students interpret quantitative, qualitative, and graphical descriptions of the same data; tell a story from a curve

**Session 6:  
Interpreting Graphs**

Students interpret different graphical shapes

**Sessions 7, 8:  
Mystery Graphs**

Students make and interpret different graphical shapes

## Concept Map for *Packages and Groups*

Students continue to build their understanding of multiplication and division, extending concepts and skills studied earlier in the year in *Arrays and Shares*. They use 300 charts, rectangular arrays, games, and calculators to begin to develop theories about finding factors of large numbers; work on learning single-digit multiplication pairs; develop and refine their strategies for solving multidigit multiplication and division problems

**INVESTIGATION 1: *Multiplication Tables*** – Students look for and use the multiplication patterns of numbers; become familiar with the multiples of larger numbers; identify factors of larger numbers

**Sessions 1, 2: Making a Multiplication Table**

Students construct multiplication tables from arrays; look for patterns in the tables

**Session 3: Multiple Plaids (Excursion)**

Students work in groups to create a set of tables for multiples 2 through 12 and then write statements about the patterns they observe

**Sessions 4, 5: Multiples of Larger Numbers**

Students skip count by two-digit numbers; find multiples of larger numbers

**INVESTIGATION 2: *Double-Digit Multiplication*** – Students use familiar landmark numbers to solve problems; partition large numbers to multiply them more easily; solve double-digit multiplication problems

**Session 1: Multiplying Two-Digit Numbers**

Students are reintroduced to multiplication cluster problems; solve problems using familiar multiplication combinations

**Sessions 2, 3: Solving and Creating Cluster Problems**

Students continue to use familiar landmark numbers to make estimates; partition numbers into smaller, more familiar numbers; solve double-digit multiplication problems

**INVESTIGATION 3: *Multiplication and Division Choices*** – Students understand how division notation can represent a variety of division situations; create a context that is representative of a division equation; use familiar landmarks to solve problems; use multiplication and division relationships to solve problems; explore factors of large numbers; find multiples

**Sessions 1, 2:  
Division Notation and Situations**

Students recognize division situations; use different types of division notation; solve division problems; relate multiplication and division equations;  
*make sense of remainders*

**Session 3: Looking More Closely at Division Problems**

Students work on division problems by grouping and by using multiples of ten; discuss strategies for keeping track of their work

**Sessions 4, 5, 6: Choice Time**

Students choose from a selection of activities that focus on solving multiplication and division problems

**Sessions 7, 8: What Are Numbers Divisible By?**

Students describe the relationships between factors and multiples; recognize the relationships among the factors of a number; develop conjectures about divisibility

**Session 9:  
Division Bingo**

Students find factors of large numbers; test conjectures for finding factors and propose new ones

**Session 10: Assessing Students' Understanding of Division (Assessment)**

Students' work focuses on identifying multiplication and division situations; using multiplication and division; finding factors of large numbers

## Concept Map for *Sunken Ships and Grid Patterns*

Students name and locate points on a coordinate grid with ordered pairs of numbers (both positive and negative); make coordinate “mystery” pictures; measure distances on a grid; discuss properties of rectangles; write rectangle procedures for the computer

[NOTE: This unit makes substantial use of the *Geo-Logo* computer program]

**INVESTIGATION 1: *Locating Houses and Ships on a Grid***— Students use positive and negative coordinates to name and locate points on grids; calculate distances on a grid based on paths along grid lines; explore numerical patterns that represent geometric situations; connect visual and numerical descriptions of distances on a grid

**Session 1: Coordinates and Distances on a Grid**

Students determine locations on a map; use coordinates to name and locate points; write coordinates as mathematicians do; find the distance between two points on a grid

**Session 2: Introducing Negative Coordinates**

Students now use negative as well as positive coordinates to find and name locations on a map; refer to points by the quadrant in which they lie

**Sessions 3, 4: Playing Sunken Ships**

Students find all locations a given distance away from a point on a grid; combine coordinates and distances on a full grid; identify areas on a grid as points  $<$  or  $>$  a particular distance; discover patterns in the lengths of paths on a grid

**Sessions 5, 6: Distances On and Off the Computer**

Students use the computer program *Geo-Logo* to draw paths on a grid; use coordinate values as information in finding the distance between two points; locate points on a grid that satisfy complex constraints

**INVESTIGATION 2: *Rectangles, Turns, and Coordinates*** – Students apply their knowledge of coordinates to locate points on a computer screen; describe geometric figures in different ways; create and apply patterns and mental arithmetic strategies to solve “turtle” geometry problems; use mirror and rotational symmetry to place rectangles on a grid and to design complex patterns of rectangles

**Session 1: Making Rectangles**

Students review their definitions of rectangles; identify their properties (four  $90^\circ$  angles; opposite sides equal in length); use *Geo-Logo* commands to write procedures for drawing rectangles

**Sessions 2, 3: Rectangles, Coordinates, and Symmetry**

Students plan a project that involves exact measurement, description of rectangles, and specification of locations; place rectangles symmetrically on a coordinate grid; find folding lines in geometric shapes

**Session 4: Properties of Rectangles**

Working on and off the computer, students discuss and use rectangle procedures with variables to draw different-shaped and -sized rectangles

**Session 5: Turns**

Students explore what happens when turns are repeated; become familiar with degrees as a common measurement for turns; estimate turn measures and understand turns of  $360^\circ$ ,  $180^\circ$ , and  $90^\circ$

**Sessions 6, 7: Turning and Repeating Rectangles (Assessment)**

Students use their knowledge of the properties of rectangles to identify rectangles and nonrectangles

**Sessions 8, 9: Designing Rectangle Patterns**

Students design complex rectangle patterns both on and off the computer

## Concept Map for *Three out of Four Like Spaghetti*

Students collect, display, describe, and compare categorical (nonnumerical) data; use fractions to describe and compare data collections; try different ways to classify collected data; notice how different classification schemes reveal different things about data; classify, display, describe, and interpret data they have collected comparing first graders' and fourth graders' career plans

**INVESTIGATION 1: *Using Fractions to Describe Data*** – Students learn to partition a group according to a rule; find familiar fractions; estimate complex fractions using familiar fractions; collect and analyze categorical data; describe data in terms of fractions; use fractions to compare data from two groups; recognize that fractions are always fractions of a particular whole

<p style="text-align: center;"><b>Session 1:</b> <b>Playing Guess My Rule</b></p> <p>Students use fraction language and notation to divide the class into categories; identify the fraction of the class in each category</p>	<p style="text-align: center;"><b>Session 2:</b> <b>Finding Familiar Fractions</b></p> <p>Students review the sizes of familiar fractions, relative to one another; understand the sizes of unfamiliar fractions by identifying which familiar fractions they are close to</p>	<p style="text-align: center;"><b>Session 3:</b> <b>Comparing Data with Familiar Fractions</b></p> <p>Students collect and organize data; represent and compare data using fractions</p>	<p style="text-align: center;"><b>Session 4:</b> <b>Using Fractions to Compare Data</b></p> <p>Students compare data from two different-sized groups; use data and fractions in word problems</p>
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**INVESTIGATION 2: *Looking at Data in Categories*** – Students collect, record, and organize categorical data; make judgments about sets of categories; redefine categories to accommodate additional information; represent categorical data in different ways (including use of bar graphs); describe categorical data; use fractions to compare categorical data from two groups

<p style="text-align: center;"><b>Session 1:</b> <b>Games We Play</b></p> <p>Students collect, record, organize, and graph categorical data</p>	<p style="text-align: center;"><b>Session 2:</b> <b>More Games, and What Have We Eaten?</b></p> <p>Students continue work on categorizing collected data and then graphing those data</p>	<p style="text-align: center;"><b>Session 3:</b> <b>What Do You Want to Be When You Grow Up?</b></p> <p>Students collect and organize data; think about possible differences in categorical variables between two groups</p>
<p style="text-align: center;"><b>Session 4:</b> <b>Organizing Some First and Fourth Grade Data</b></p> <p>Students construct categories for categorical data; compare different sets of categories; communicate their system for categorizing the data; look at the same data categorized in a few different ways</p>	<p style="text-align: center;"><b>Sessions 5, 6, 7:</b> <b>Making Comparisons with All the Data</b></p> <p>Students adjust categories to accommodate all the data; graph the data; use fractions to compare first and fourth graders' career choices; communicate their findings about the collected data; compare their work with alternative approaches to classifying the data</p>	