BRIDGES IN MATHEMATICS 5

GRADE 5 SUPPLEMENT

Set D2 Measurement: Volume

Includes

Skills & Concepts

- ★ determine volume by finding the total number of same-sized units of volume that fill a three-dimensional shape without gaps or overlaps
- ★ understand a cube that is one unit on an edge is the standard unit for measuring volume
- ★ select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume
- ★ measure necessary attributes of shapes to use volume formulas to solve problems

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Bridges in Mathematics Grade 5 Supplement
Set D2 Measurement: Volume

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Bridges in Mathematics is a standards-based K–5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates the Number Corner, a collection of daily skill-building activities for students.

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Set D2 ★ Activity 1

ACTIVITY

Introducing Volume

Overview

In this activity, students move toward increasingly efficient methods of finding the volume of cubes and rectangular solids.

Skills & Concepts

- ★ determine volume by finding the total number of same-sized units of volume that fill a three-dimensional shape without gaps or overlaps
- ★ understand a cube that is one unit on an edge is the standard unit for measuring volume
- ★ select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume
- ★ measure necessary attributes of shapes to use volume formulas to solve problems

You'll need

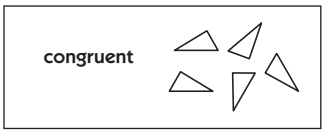
- ★ Cubes & Rectangular Solids (page D2.4, run 1 copy on a transparency)
- ★ Paper Box Pattern (page D2.5, run a class set)
- ★ centimeter cubes (class set)
- ★ scissors
- ★ scotch tape
- ★ rulers (class set)
- ★ Student Math Journals
- ★ Word Resource Cards (*congruent, edge, face, parallel, perpendicular, vertex*)

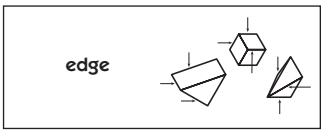
Advance Preparation Display the Word Resource Cards where students can see them before conducting the activity.

Instructions for Introducing Volume

1. Give students each a centimeter cube and allow several minutes for them to record as many observations as they can about the cube in their math journals. Call their attention to the Word Resource Cards before they start writing and challenge them to include at least 3 of the words in their observations.

2. Have them pair-share their observations, and then call for whole-group sharing. Record some of their observations at the top of the Cubes and Rectangular Solids overhead, keeping the rest of the transparency covered for now. If it doesn't come up in the discussion, ask students to find examples of parallel, perpendicular, and congruent edges and faces as they examine their cubes.

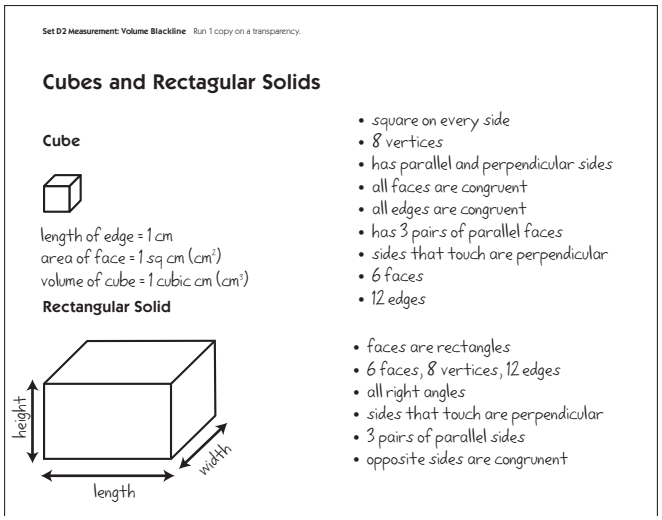




3. Ask students to estimate the length of one of the edges of their cube. Then have a volunteer measure to confirm that each edge is 1 centimeter. Next, ask students to determine the area of one of the cube's faces. Finally, explain that because their cube is 1 centimeter long, wide, and high, it is called a *cubic centimeter*. Just as centimeters are used to measure length and square centimeters are used to measure area, *cubic centimeters* are used to measure *volume*. Add this information to the overhead, along with the abbreviations for each measure.

Activity 1 Introducing Volume (cont.)

4. Next, reveal the picture of the rectangular solid on the overhead. Have students write at least 3 observations about this figure in their journals. Then invite volunteers to share their observations with the class as you record at the overhead. After you've recorded 8–10 observations, work with input from the students to label all 3 dimensions of the solid: length, width, and height.



5. Give each table a good supply of centimeter cubes. Ask each student to build several different rectangular solids that have a volume of exactly 12 cubic centimeters. Be sure they understand that their constructions have to be solidly filled in, without gaps or holes between cubes. Ask them to share and compare their constructions as they're working.

6. After a few minutes, call a halt to the construction process. Ask several volunteers to describe their constructions by length, width, and height. Record each description at the board, along with an equation to confirm that the total is 12 cubic centimeters.

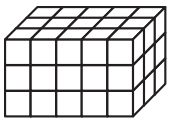
Rectangular Solids with Volume = 12 cm^3

Length = 2 cm	Length = 6 cm	Length = 12 cm
Width = 2 cm	Width = 2cm	Width = 1 cm
Height = 3 cm	Height = 1cm	Height = 1 cm
$2 \times 2 \times 3 = 12\ cm^3$	$6 \times 2 \times 1 = 12\ cm^3$	$12 \times 1 \times 1 = 12\ cm^3$

7. Now reveal the rectangular solid at the bottom of the overhead. Ask students to replicate it with their cubes and determine its volume without counting every cube one by one. As they finish, invite volunteers to share their strategies with the class, as you record at the overhead. If it doesn't come from one of the students, ask them what would happen if you multiplied length × width × height. Would it result in the same answer they've shared? Why or why not? Press them to explain their thinking and then work with their input to write the equation and solve the multiplication problem.

Activity 1 Introducing Volume (cont.)

Build this rectangular solid with your centimeter cubes. Find the volume *without* counting each cube 1 by 1.



$15 + 15 + 15 = 45\ cm^3$

$9 \times 5 = 45\ cm^3$

counted by rows of 5 ($5 \times 9 = 45\ cm^3$)

$18 + 18 = 36\ \ 36 + 9 = 45\ cm^3$

length x width x height

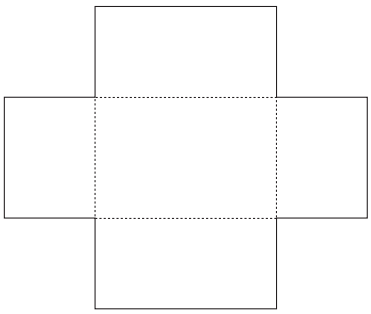
$5 \times 3 \times 3 = 45\ cm^3$

8. Ask students to clear their cubes to the side for now and get out their scissors. Give each student a copy of the Paper Box Pattern and supply each table with some scotch tape. Have them cut, fold, and tape their paper patterns to make a box. Ask early finishers to help others near them.

Set D2 Measurement: Volume Blackline Run a class set.

Paper Box Pattern

Cut out this pattern. Fold along the dashed lines and tape to make a box.



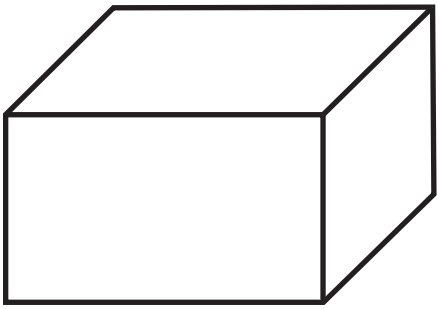
9. When everyone has finished, ask students to estimate the volume of the box. How many centimeter cubes do they think it will take to fill the box completely? Record some of their estimates on the board. Then challenge them to work in pairs to determine the actual volume of the box *without* filling it to the top with cubes, dumping them out, and counting them one by one. As they finish, have them record their solution in their journal, along with a detailed description of their strategy.

10. Toward the end of the period, reconvene the class. Ask volunteers to share their strategies and solutions with the class. If the idea of measuring the dimensions of the box and multiplying them doesn't come from one of the students, ask them to get out their rulers and try it. Does it result in the same solution they got using other methods? Why? (Students should find that the taped box holds 54 centimeter cubes. It is 6 centimeters long, 3 centimeters wide, and 3 centimeters high. $6 \times 3 \times 3 = 54\ cm^3$.)

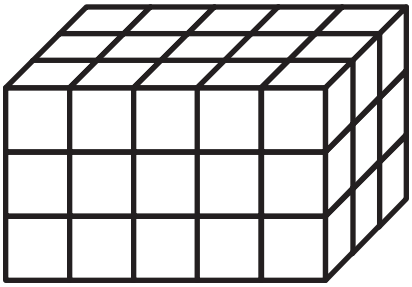
Cubes & Rectagular Solids

Cube

Rectangular Solid

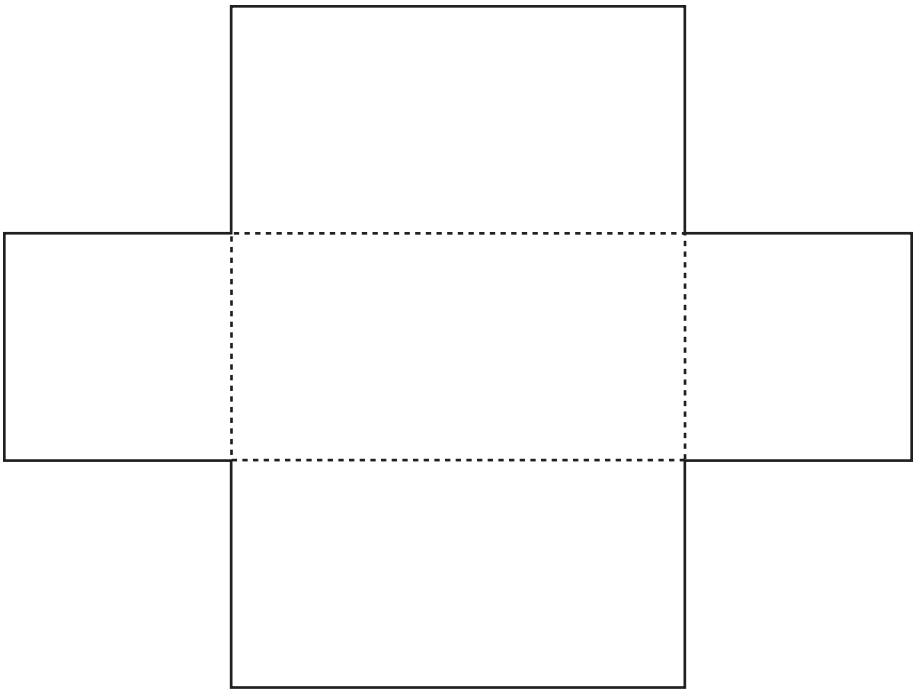


Build this rectangular solid with your centimeter cubes. Find the volume *without* counting each cube 1 by 1.



Paper Box Pattern

Cut out this pattern. Fold along the dashed lines and tape to make a box.



Set D2 ★ Activity 2

ACTIVITY

More Paper Boxes

Overview

Using paper boxes and centimeter cubes, students work together to generate efficient methods, including the standard formulas, for finding the volume of cubes and rectangular solids.

Skills & Concepts

★ determine volume by finding the total number of same-sized units of volume that fill a three-dimensional shape without gaps or overlaps

★ select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume

★ measure necessary attributes of shapes to use volume formulas to solve problems

You'll need

★ More Paper Boxes (page D2.10, half-class set, plus a few extra)

★ centimeter cubes (class set)

★ scissors

★ scotch tape

★ rulers (class set)

★ Student Math Journals

★ *Counting on Frank* by Rod Clement (optional)

Recommended Timing

Anytime after Set D2 Activity 1

Instructions for More Paper Boxes

1. Ask students to pair up, or assign partners. Students will need their rulers, scissors, and journals for this activity. Give each pair a copy of More Paper Boxes, along with some scotch tape. Have them cut out and tape together Box A, leaving Boxes B and C uncut for now. Ask early finishers to help others nearby.

2. When most students have finished constructing Box A, ask them to estimate how many centimeter cubes it will take to fill the box completely. Have them each record an estimate in their journals. Then ask volunteers to share and explain their estimates as you record at the board.

> ***Lauren*** *It looks like it's going to take about 10 to fill the bottom, and it's about 3 cubes high, so I think 30 cubes will do it.*
>
> ***Tonio*** *I say 40 because it's maybe 10 on the bottom and 4 up. That would be 4 × 10, so that's 40.*
>
> ***Marisa*** *I said 54 cubes because it looks like it's 3 across and maybe 6 long. That's 18. I think it's going to be 3 layers high, so I multiplied 3 × 18 to get 54.*

3. Distribute centimeter cubes and ask student pairs to find the actual volume of Box A. Explain that they can use any method they want except filling the box completely, dumping out the cubes, and counting them one by one. As they finish, have them record their answer, along with a description of their strategy in their journal.

Activity 2 More Paper Boxes (cont.)

4. After they've had a few minutes to work, ask volunteers to share their solutions and strategies with the class.

> ***Carter*** *It took 21 cubes to cover the bottom of the box. Then we stacked cubes in one corner to find out how high the box was. It was 4 cubes up, so we said 4 × 21 is 84 cubes.*
>
> ***Abby*** *We just used the cubes to make kind of an outline inside the box. It was 7 on the long side and 3 on the short side, so we knew the first layer would be 21. Then we went up one corner like Carter and Xavier, and it was 4. Then we knew it was 84 cubic centimeters because 4 × 21 is 84.*

5. If the idea of measuring the dimensions of the box and multiplying them doesn't come from the students, ask them to get out their rulers and try it. Does this strategy result in the same solution they got using other methods? Why? Work with class input to record an equation that matches what they just did: $7 \times 3 \times 4 = 84$ cm^3.

6. Ask students to cut out and tape together Box B and record an estimate of the volume in their journals. As they're working, collect the centimeter cubes. When most have finished, ask volunteers to share their estimates as you record at the board. Then challenge students to find the actual volume of the box using their rulers instead of cubes. Have them record the answer, along with any computations they made, in their journal.

7. After they've had some time to work, ask volunteers to share their solutions and strategies with the class. Then work with input from the class to write a general formula for finding the volume of a rectangular solid (length × width × height = volume), along with an equation for Box B ($6 \times 4 \times 2 = 48$ cm^3). Have students record this information in their journals.

8. Now tell them that some fifth graders in another class said they thought they could find the volume of Box C without cutting and taping it together. Do your students agree with these fifth graders? Why or why not? Have them pair-share their responses and then ask volunteers to share their thinking with the class.

> ***Students*** *We said you could do it by just using a ruler, but you should cut out the box and put it together first.*
> *We think they're right. It looks like it's going to be a cube, so if you just measured one edge, you could figure it out.*

9. Ask students to measure one or more edges of the uncut box to help make as accurate an estimate as possible. Have them record their estimate, along with an explanation in their journal. (If they're sure their estimate matches the actual volume, that's fine.)

10. After a few volunteers have shared and explained their estimates, ask students to cut out and tape together Box C. Have them measure it to determine the actual volume, and record the answer, along with any calculations they made, in their journals.

11. Have volunteers share and explain their solutions and strategies. Was it possible to determine the volume of the figure by measuring only 1 edge? Why or why not? Would it have been possible to find the answer without cutting and taping the cube? Why or why not? Then have students write an equation for the volume of Box C ($4 \times 4 \times 4 = 64$ cm^3) in their journals.

Activity 2 More Paper Boxes (cont.)

Extensions

- Explain that because the length, width, and height of a cube are all equal, mathematicians generally represent $s \times s \times s$ by s^3. Using this notation, the volume of a cube is s^3 where s is the length of one edge of the cube.

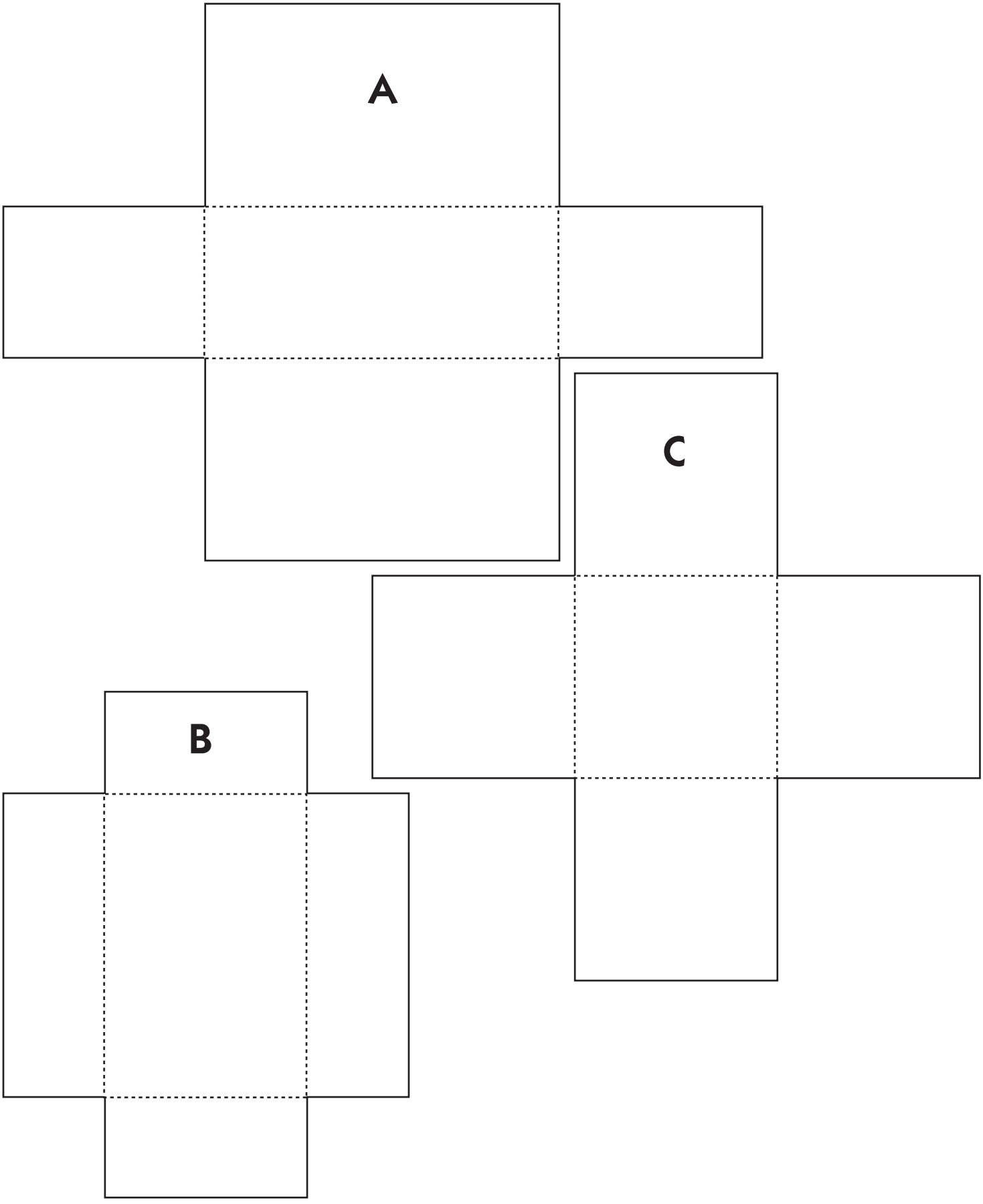
  $$s \times s \times s = s^3$$

  Record this at the board, and ask students to compare it to the formula for finding the volume of a rectangular prism. How are the two alike? How are they different? Ask them to record the general formula for finding the volume of a cube in their journals.
- Have volunteers use lightweight cardboard and tape to construct a cubic inch and a cubic foot, and share them with the class. Ask students to list in their journals some of the things they'd measure in cubic inches and some of the things they'd measure in cubic feet.
- Read *Counting on Frank* by Rod Clement before or after this session.

INDEPENDENT WORKSHEET

See Set D2 Independent Worksheets 1 and 2 for more practice selecting and using appropriate units and formulas to determine length, area, and volume.

More Paper Boxes



NAME _____ **DATE** _____

Set D2 ★ Independent Worksheet 1

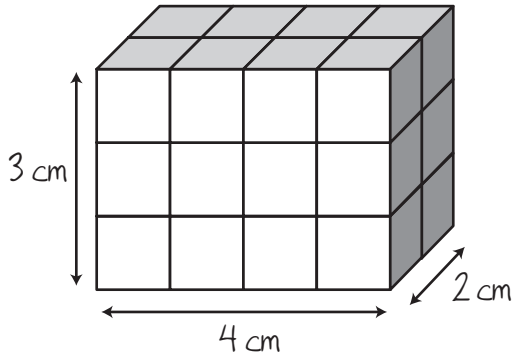
INDEPENDENT WORKSHEET

Volume Review

Volume is the measure of the space occupied by a 3-dimensional object. Volume is measured in cubes of a given size, such as cubic centimeters, cubic inches and cubic feet.

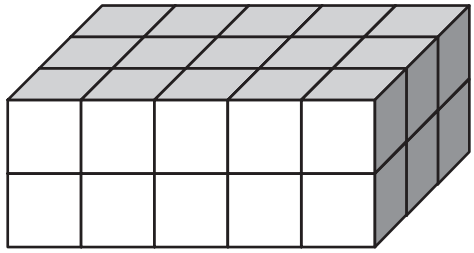
1 Each of the rectangular solids below was built with centimeter cubes. Label each with its dimensions (length, width, and height) and find the volume. Show your work.

example



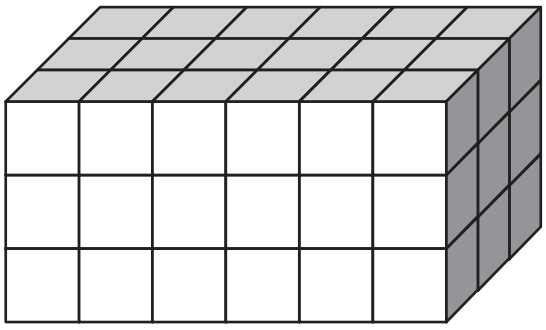
Volume $4 \times 2 \times 3 = 24$ cubic cm (or cm^3)

a



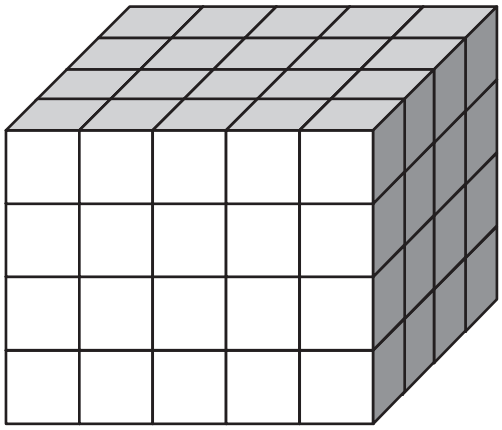
Volume

b



Volume

c



Volume

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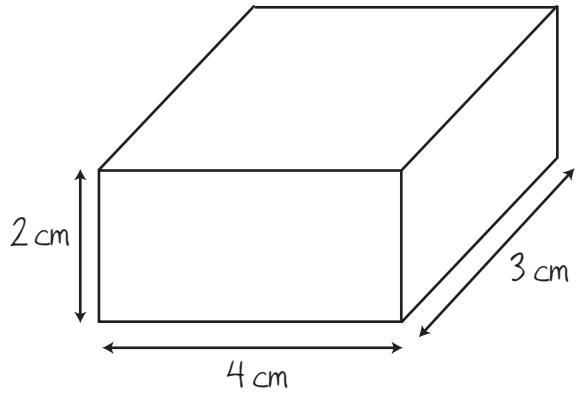
Independent Worksheet 1 Volume Review (cont.)

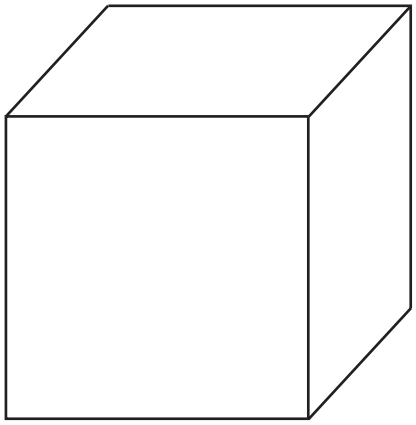
2 Use the centimeter side of your ruler to measure the dimensions of each rectangular solid below. Then find its volume. Show your work.

example



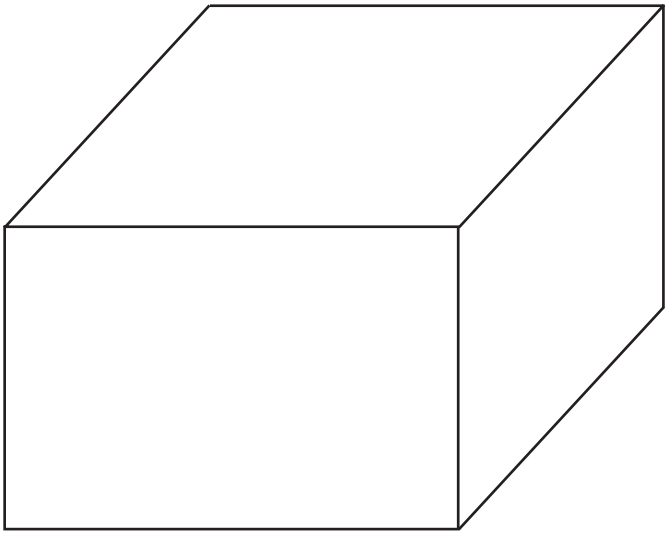
Volume $4 \times 3 \times 2 = 24$ cubic cm (or cm^3)

a



Volume

b



Volume

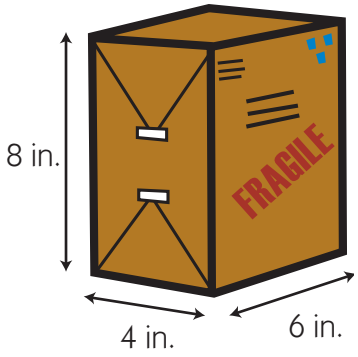
(Continued on next page.)

NAME _____ **DATE** _____

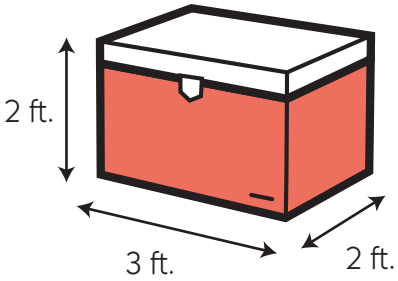
Independent Worksheet 1 Volume Review (cont.)

3 Miguel says you only need to measure one edge of a cube to find its volume. Do you agree with him? Why or why not? Use numbers, labeled sketches, and words to explain your answer.

4 Mia has already measured the dimensions of this packing box. Help her find the volume. Show your work.

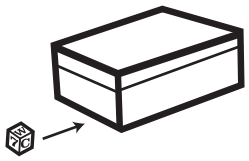


5 Brandon is going on a fishing trip with his family. He wants to find the volume of the family's ice chest. Which expression should he use?



- ○ 2×3
- ○ $3 \times 2 \times 2$
- ○ $3 + 2 + 2$
- ○ $(3 \times 2) - 2$

6 Jeff's little brother is trying to find out how many alphabet blocks will fit into a shoebox. He is measuring:



- ○ the volume of the shoebox
- ○ the area of the shoebox
- ○ the length of the shoebox

(Continued on back.)

Independent Worksheet 1 Volume Review (cont.)

7 Which of these situations is about volume?

○ determining the amount of fencing it takes to go around a square garden

○ determining how many tiles it will take to cover the kitchen floor

○ determining how many rectangular containers of food will fit into a freezer

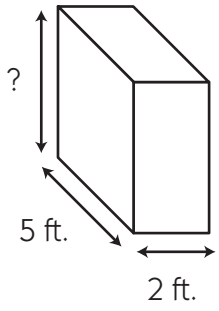
8 Vanesa wants to find the volume of her lunchbox. Which of these units should she use?

○ cubic feet

○ cubic inches

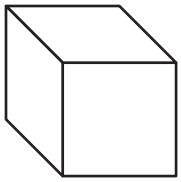
○ cubic yards

9 The volume of this rectangular solid is 40 cubic feet. What is its height? Show your work.



CHALLENGE

10 The volume of this cube is 125 cubic inches. What is the length of each edge? Show your work.



NAME _____ **DATE** _____

Set D2 ★ Independent Worksheet 2

INDEPENDENT WORKSHEET

The Camping Trip

The Gomez family is going on a camping trip next week. There are 4 people in the family: Mr. and Mrs. Gomez and the 11-year-old twins, Ramon and Dora. Help them do some planning for their trip. Circle a correct answer to each question below.

1 Mrs. Gomez wants to cut a piece of rope that's long enough to dry the family's laundry on every day. Which of these units should she use to measure the rope?

inches feet yards miles

2 Mr. Gomez wants to figure out how far they'll have to drive to get to the campsite. He already knows that it will take about a day to get there. Which of these units should he use?

inches feet yards miles

3 The shoelaces on Ramon's tennis shoes are almost worn out. He has to measure them so he gets the right length at the store. Which of these units should he use?

millimeters centimeters meters kilometers

4 Mrs. Gomez says it's going to be a 3-minute walk from their tent to the lake. Dora wants to measure the distance when they get there. Which of these units should she use?

millimeters centimeters meters kilometers

5 Ramon wants to find the area of his sleeping bag to see how much room he'll have in the family's tent. Which of these units should he use?

square inches square feet square yards square miles

Independent Worksheet 2 The Camping Trip (cont.)

6 Which formula should Ramon use to find the area of his sleeping bag?

Area = Length + Width Area = Length × Width Area = Length ÷ Width

7 Dora says when they get there, she's going to measure the area of their campsite. Mrs. Gomez says the campsite is big enough for their car, their tent, their picnic table and chairs, and their campfire, with a little room left over. Which of these units should she use?

square inches square feet square yards square miles

8 Which formula should Dora use to find the area of the campsite?

$A = (2 \times l) + (2 \times w)$ $A = (3 \times l) - (2 \times w)$ $A = l \times w$

9 Mr. Gomez wants to find the volume of the family car trunk so he'll know how much luggage will fit back there. Which of these units should he use?

cubic inches cubic feet cubic yards

10 Ramon wants to measure the volume of a shoebox to find out how many CD's he can fit into it for the trip. Which of these units should he use?

cubic inches cubic feet cubic yards

11 Dora is going to collect tiny pebbles at the lake. She wants to measure the volume of a metal band-aid box to keep them in. Which of these units should she use?

cubic centimeters cubic meters cubic kilometers